

# Le molte facce della matematica (giochiamo con gli esaflexagoni)

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UNIVERSITÀ  
CATTOLICA  
del Sacro Cuore

Celebration of Mind, 21 ottobre 2015

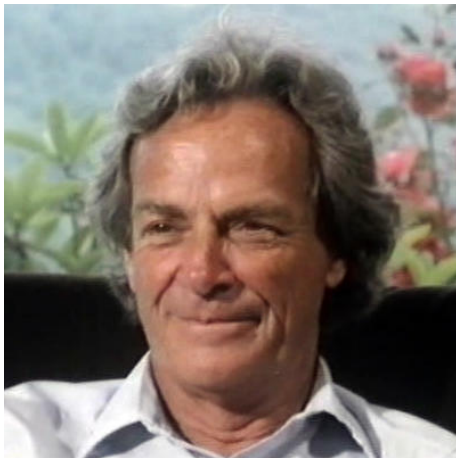
# Tutto comincia nel 1939. . .



Arthur Stone (30 settembre 1916 – 6 agosto 2000)

Arthur Stone, matematico britannico, scoprì per caso i *flexagoni* a Princeton nel 1939.

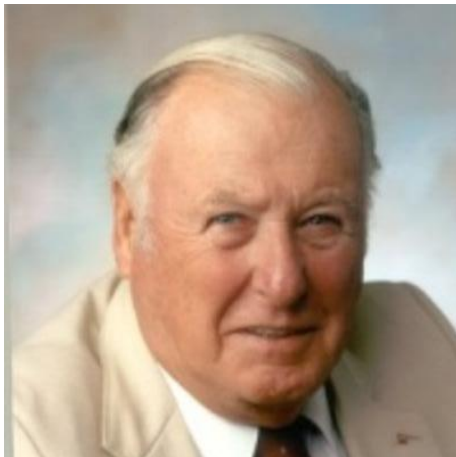
# Tutto comincia nel 1939...



Richard Feynman (11 maggio 1918 – 15 febbraio 1988)

Richard P. Feynman, fisico statunitense, premio Nobel, fece parte del *Princeton Flexagon Committee*, nato per studiare i flexagoni.

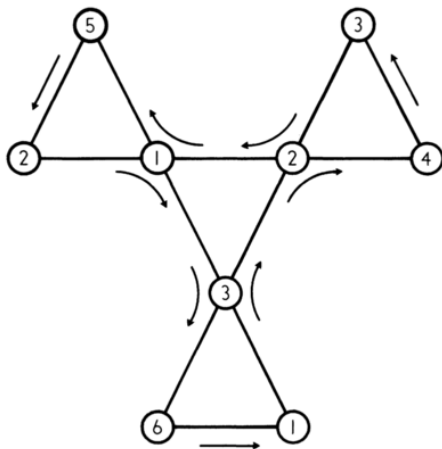
# Tutto comincia nel 1939. . .



John Tukey (16 giugno 1915 – 26 luglio 2000)

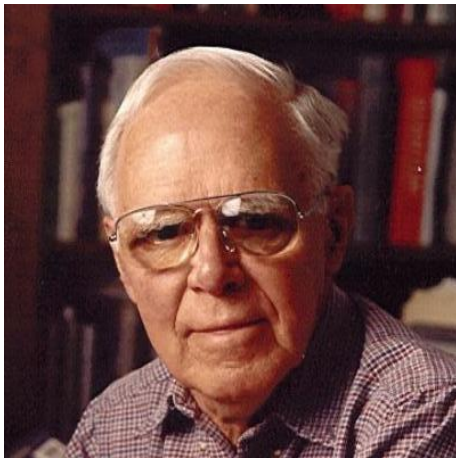
John W. Tukey, matematico e statistico statunitense, noto per aver inventato la FFT, fece parte del *Princeton Flexagon Committee*.

# Tutto comincia nel 1939...



Bryant Tuckerman (28 novembre 1915 – 19 maggio 2002)

Louis Bryant Tuckerman III, matematico e informatico statunitense, uno degli sviluppatori del DES (Data Encryption Standards), fece parte del *Princeton Flexagon Committee*.



Martin Gardner (21 ottobre 1914 – 22 maggio 2010)

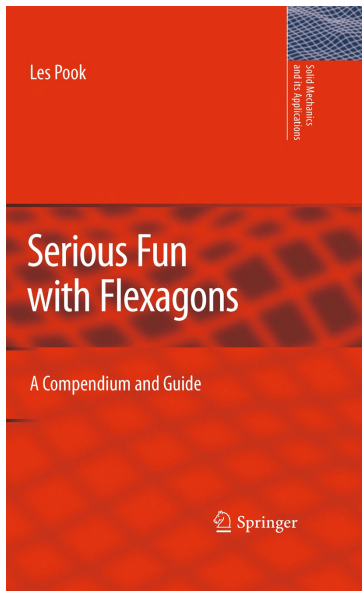
Martin Gardner. Il primo articolo della rubrica *Mathematical Games* del dicembre 1956 su *Scientific American* trattava dei flexagoni: “*Flexagons, in which strips of paper are used to make hexagonal figures with unusual properties*”

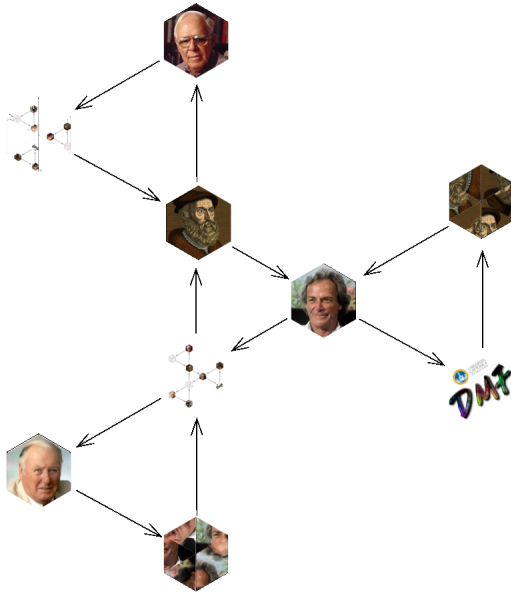


**“TUCKERMAN’S TRAVERSE”** exposes all six faces of a hexahexaflexagon in 12 flexes. Here the numbers of the flexagon at the top of the page have been replaced by geometrical figures in the same pattern. Faces 1, 2 and 3 turn up three times as often as faces 4, 5 and 6.

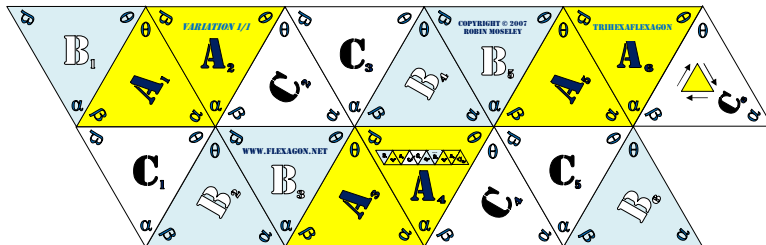








## TRIHEXAFLEXAGON VARIATION 1/1

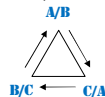


### TRIHEXAFLEXAGON VARIATION 1/1



Fold in half along the horizontal middle line. Before gluing, you may want to pre crease all triangle lines back and forth. Once folded and glued into a horizontal strip of triangles, fold together the two pairs of C triangles. Now adjust the ends so that the reverse side of the top left "B" triangle will fold over and paste onto the back side of the top right "C" triangle. All the yellow A triangles will be on one face of the flexagon when completed. Note that the flexagon has a uniform thickness of 2 sheets of paper when completed.

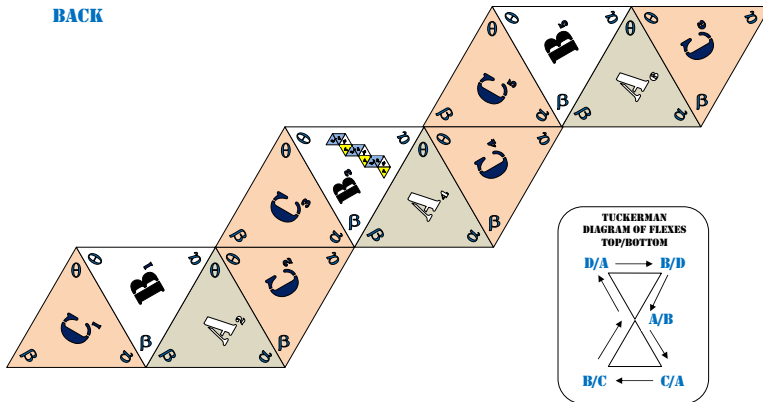
### TUCKERMAN DIAGRAM OF FLEXES TOP/BOTTOM

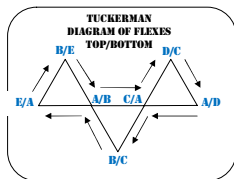


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WWW.FLEXAGON.NET

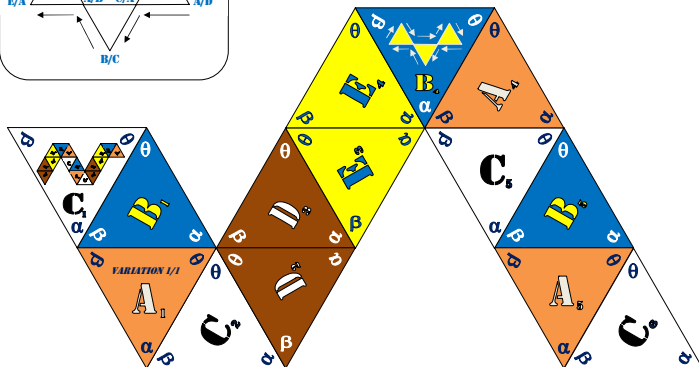
## TETRAHEXAFLEXAGON VARIATION 1/1

BACK

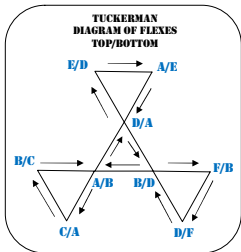




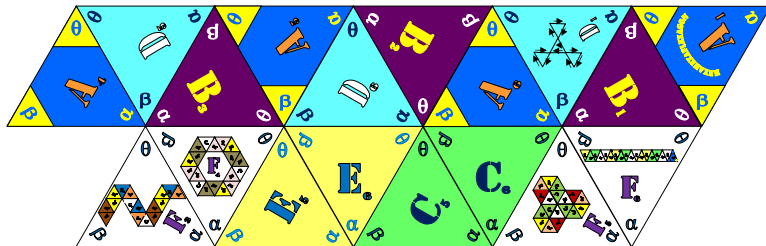
## PENTAHEXAFLEXAGON VARIATION 1/1



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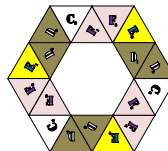
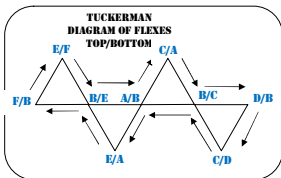


## HEXAHEXAFLEXAGON VARIATION 1/3



## HEXAHEXAFLEXAGON VARIATION 2/3

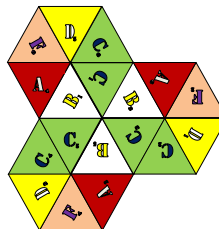
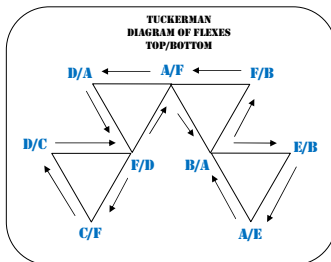
This is a 6 sided template for a 6 sided hexagon with 6 faces! I find it can be easier (especially if you use a fast drying glue) to leave the inside uncut until after gluing together. You can pre crease all triangles before and after gluing. Glue the backside such that the reverse side of the  $F_2$  and  $E_3$  triangles are not covered until the flexagon is assembled.  $F_1$  will be behind  $E_2$ . Once glued, cut out the middle and crease all the lines back and forth. Now fold together the three pairs of Fs. You will have a pattern as for a Pentahehexaflexagon. Fold together the three pairs of D triangles, it will look like the pattern for the tetrahexaflexagon. Now fold the three pairs of C triangles and then the last two E triangles. The back of the  $F_2$  and  $E_3$  triangles will be on the front and back faces. Glue the  $A_1/B_2$  triangle unit to the back of the  $F_2$  and  $E_2$  triangles. All the As should be on one face of the completed flexagon, the Bs on the back and the flexagon will have a uniform thickness of 2 sheets of paper. This is the second of 3 variations of the hexahehexaflexagon.



## HEXAHEXAFLEXAGON VARIATION 3/3

This is a great flexagon, easy to fold, glue and the final model is excellent. The Tuckerman diagram is also rather different than the ones we have seen so far. I find it easier to cut out the outside lines and leave the inside uncut until after gluing together. You can easily pre crease all triangles before and after gluing. Glue the two sides together such that  $C_2$  will be behind  $F_3$  and  $A_6$  behind  $E_4$ . Once glued, cut out the middle lines and crease all the lines back and forth. Now fold together the three pairs of Cs. You will have a pattern as for a Pentahexaflexagon. Fold together the three pairs of D triangles, it will look like the pattern for the tetrahexaflexagon. Now fold the three pairs of E triangles and then the last three F triangles. The back of the  $F_2$  and  $D_1$  triangles will be on the front and back faces. Glue the  $A_1/B_4$  triangle unit such that the As should be on one face of the completed flexagon, the Bs on the back. The flexagon will have a uniform thickness of 2 sheets of paper. This is the third of 3 variations of the hexahexaflexagon.

Short sequence for folding Variation 3 - Cs-Ds-Es-Fs

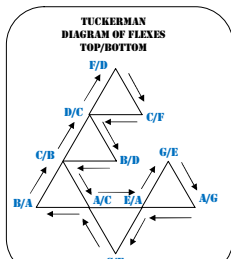




## THE SNAKE HEPTAHXAFLEXAGON VARIATION 1/4

This flexagon has 7 faces. You can pre crease all triangles before and after gluing. Glue the backside such that the reverse side of the  $D_1$  and  $F_1$  triangles are not covered until the flexagon is assembled.  $F_1$  will be behind  $D_1$  once assembled.  $F_2$  will be behind  $C_2$  when the two halves are assembled. After gluing crease all the lines back and forth. Now fold together the three pairs of  $G$ s. You will have a pattern as for a hexahexaflexagon. Fold together the three pairs of  $F$  triangles, it will look like the pattern for the pentahexaflexagon. Now fold the three pairs of  $E$  triangles and then the  $A$  triangles and finally the  $B$  triangles. The back of the  $F_1$  and  $D_1$  triangles can now be glued together to lock the flexagon in place. All the  $C$ s should be on one face of the completed flexagon, the  $D$ s on the back and the flexagon will have a uniform thickness of 2 sheets of paper. I recommend gluing the halves together two to three triangles at a time. Do not try to do it all at once, you will get a crumpled sticky mess. This is the first of 4 variations of the heptahexaflexagon.

Quick Folding sequence: 3Gs – 3Fs – 3Es – 3As – 3Bs



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## HEPTAHXAFLEXAGON

### VARIATION 2&3/4

Crease all triangles before and after gluing. Fold both parts in half and then glue each together leaving the triangles without a matching back untouched. Now attach the two parts together by gluing the back of the  $D_1$  triangle to the back of the  $E_6$  triangle. Once glued, crease all the lines back and forth again. Trim off any imperfections with sharp scissors. Now fold one or the other variation as follows:

Main Variation "B":

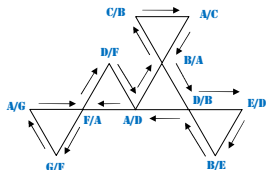
G1-G6, G2-g3, G4-G5, F2-F5, E2-E3, C6-C1, F2-F3, E1-E6, C4-C5, F1-F6, E4-E5, C2-C3, D5-D6, D1-D2, Flip D4 up on top and Glue D4 to F5. The As will be on one face and Bs on the other. Adjust all the triangles during folding to be exactly on top of each other. You will like the result! Each Letter will have a face where they radiate from the center in numerical order.

Second Variation "C" with Weekday faces:

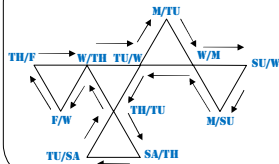
D6-B2, F3-C6, Sat2-Sat3, Sat4-Sat5, Sat1-Sat6, G1-G6, G2-G3, G4-G5, Fri1-Fri6, Fri4-Fri5, Fri2-Fri3, Mon2-Mon3, Mon4-Mon5, Mon6-Mon1, Wed1-Wed2, Wed3-Wed4, Wed5-Wed6, Glue Thurs5-Sat4. Each day of the week will have a face with the name of the week radiating from the center in numerical order.

Note how the opposite variation Letters or Weeks twist and turn on the faces. Challenge: Can you V-Flex from one variation to the other? (You will have to look up V-flexing on the Internet).

**TUCKERMAN DIAGRAM OF FLEXES - TOP/BOTTOM  
MAIN VARIATION B**



**TUCKERMAN DIAGRAM OF FLEXES - TOP/BOTTOM  
WEEKDAY VARIATION C**



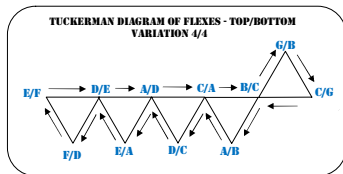
## HEPTAHEXAFLEXAGON

### VARIATION 4/4 – FIRE & WOOD

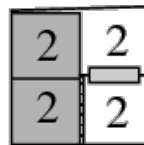
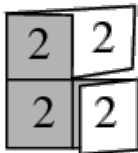
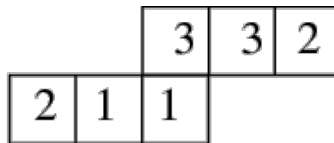
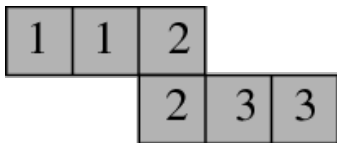
Crease all triangles before and after gluing. Align the front and back pieces so that the triangles line up as indicated in the above drawings using the indicated notation above to determine front and back triangles. Once glued, crease all the lines back and forth again. Trim off any imperfections with sharp scissors. Now fold as follows:

Fold all pairs of G triangles. Fold together; F5-F6, F1-F2. Now fold all three pairs of B triangles, B1-B2, B3-B4, B5-B6. Now fold; E5-E6, E3-E4, C2-C3, C4-C5, C1-C6. Place E1 face to face with E2 and then F4 face to face with F3. Now glue the back of F4 to the back of D1. The As will be on one face and Ds on the other. Adjust all the triangles during initial folding to be exactly on top of each other.

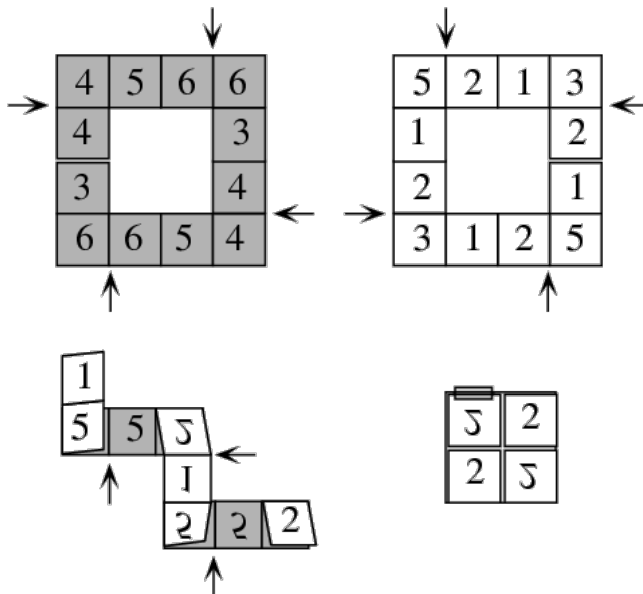
Note if you start with the E face on top and F on the bottom, you can flex 5 times before switching corners.



# Altri flexagoni: i tetraflexagoni

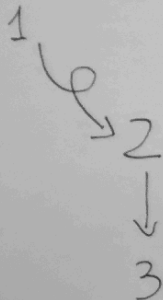


# Altri flexagoni: i tetraflexagoni

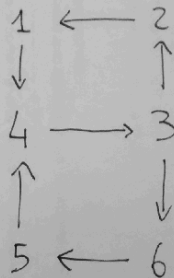


# Tetraflexagoni

TRIATETRAFLEXAGONO



ESATETRAFLEXAGONO



# Tetraflexagoni

